Example 1 (Empty $\max_{\succsim}$)**.** Consider $X = \mathbb{R}$ and $\succsim$ is more is better and $Y = (0, 1)$.

$$\max_{\succsim} Y = \emptyset.$$

Example 2 (Transitivity, quasi-transitivity, Acyclicity)**.** We give two examples. First example satisfies quasi-transitivity but fails to transitivity, second example satisfies acyclicity but fails to quasi-transitivity.

(i) consider $X = \mathbb{R}$ and $\succsim_1$ such that for $x, y \in \mathbb{R}$, $x \succsim_1 y \iff x \geq y + 1$. $\succsim_1$ satisfies quasi-transitivity but not transitive;

(ii) consider $X = \{x_1, x_2, x_3\}$ and $\succsim_2 = \{(x_1, x_2), (x_2, x_3)\}$.

Example 3 (Some remark)**.** Notice that for acyclicity, "not $x_n \succ x_1$" does not imply that $x_1 \succsim x_n$, the reason is $\succsim$ may not be complete.

Example 4 (Satisfying contraction but violating expansion)**.** Let $X = \{a, b, c\}$ and consider the following choice function.

Y	$C(Y)$
ab	ab
bc	bc
ac	ac
abc	a

This choice function violates expansion, because $b \in C(\{a, b\})$ and $b \in C(\{b, c\})$, however $b \notin C(\{a, b, c\})$.

Example 5 (Satisfying strong expansion but violating contraction)**.** Let $X = \{a, b, c\}$ and consider the following choice function.

Y	$C(Y)$
ab	a
bc	b
ac	c
abc	abc

This choice function violates contraction, because $a \in C(\{a, b, c\})$ however, $a \notin C(\{a, c\})$.

Example 6. The notion of WARP is equivalent to the following condition: for all $Y, Z \in \mathcal{M}(X)$ and $x, y \in Y \cap Z$,

$$x \in C(Y) \text{ and } y \in C(Z) \text{ implies } x \in C(Z).$$

Proof. Proof breaks into two parts.

(i) "→": as $(Y \cap Z) \subset Y$ and $\{x\} \subset \big(C(Y) \cap (Y \cap Z)\big)$, hence WARP implies $C(Y \cap Z) = C(Y) \cap (Y \cap Z)$. We can conclude: $x \in C(Y \cap Z)$. Applying $Y \cap Z$ and Z with WARP, we finally get: $C(Y \cap Z) = C(Z) \cap (Y \cup Z)$. Hence, $x \in C(Z)$.

(ii) "←": let $Z \subset Y$ and $C(Y) \cap Z \neq \emptyset$.

  (i) $\forall x \in C(Z)$, we have: $x \in Z = Z \cap Y$, therefore $x \in C(Y)$, which implies $C(Z) \subset C(Y) \cap Z$;

  (ii) $\forall x \in C(Y) \cap Z$, we have: $x \in Z \cap Y$ and $x \in C(Y)$, therefore $x \in C(Z)$, which implies $C(Y) \cap Z \subset C(Z)$.

  Therefore, $C(Y) \cap Z = C(Z)$.

□

Example 7 (Rationalizable but not by quasi-transitive relation)**.** We construct a relation that is rationalizable but not by a quasi-transitive relation.

Y	$C(Y)$
ab	b
ac	ac
bc	c
abc	c

Therefore, $b \succ_C a$ and $a \sim_C c$ and $c \succ_C b$, which clearly violates quasi-transitivity.

Example 8 (Some observations)**.** We have the following two observations.

(i) every preference is semi-order;

(ii) semi-order is quasi-transitive, however not every quasi-transitive relation is semi-order.

Following two examples illustrating the converse of observations above.

(i) not every semi-order is preference. Consider $X = \{x, y, z\}$ and $x \sim y \sim z$ and $x \succ z$. It's semi-order but not a preference.

(ii) not every quasi-transitive preference is semi-order. Consider $X = \{x, y, z, w\}$ with $x \succ y$ and $z \succ w$ and indifference otherwise. Clearly, it's quasi-transitive but not semi-order.

Example 9 (Independence of vNM3)**.** lexicographic preferences. Let $A = \{a_1, a_2, \ldots, a_n\}$ and define $\succsim$ on $\mathcal{L}(A)$ as following:

(i) $p \sim p$ for all $p \in \mathcal{L}(A)$;

(ii) $p \succ q$ if and only if there exists $k \in [n]$ so that $p(a_i) = q(a_i)$ for all $i < k$ and $p(a_k) > q(a_k)$.

Consider the following three lotteries: $p_1 = (\frac{2}{3}, \frac{1}{6}, \ldots) \succ p_2 = (\frac{1}{3}, \frac{1}{2}, \ldots) \succ p_3 = (\frac{1}{3}, \frac{1}{3}, \ldots)$. However any convex combination of p_1 and p_3 is strictly preferred to p_2.

Example 10 (Independence of F3)**.** We present an example violating De Finetti's axiom 3 but remaining satisfying other axioms. Let $S = \{s_1, \ldots, s_n\}$ for some $n \in \mathcal{N}$. Define $\succsim$ by letting, $f \sim g$ if and only if $f = g$ and $f \succ g$ if and only if there is a $k \in [n]$ such that $f(s_i) = g(s_i)$ for all $i < k$ and $f(s_k) > g(s_k)$. Let's consider for the case when $n = 2$ and $f = (1, 1)$. Then, $U_{\succsim}(f) = \{(x, y) : x > 1 \text{ or } x = 1, y \geq 1\}$ which is not open.

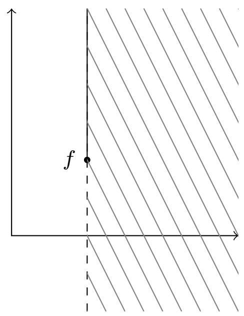

Figure 1: $U_{\succsim}(f)$